# QUAN 203 - 2021

**Tutorial 8:** try, scan and upload these exercises to Blackboard before attending your tutorial in Week 11. Those submitted by the notified deadline will be scored and contribute towards the Tutorial Assignment component of your final mark.

1. Q4 of Tutorial Assignment 7. If you are happy with your submitted answer from last week, you may either rescan it, or simply refer your tutor back to that submission.

2. Q5 of Tutorial Assignment 7. As above...

3. Show that $cov(\mathbf{X}) = E(\mathbf{X}\mathbf{X}') - \boldsymbol{\mu}\boldsymbol{\mu}'$ where $\boldsymbol{\mu} = E(\mathbf{X})$.

4. In the standard bivariate model, show that the $(i, j)$th term of $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ is
$$h_{ij} = \frac{1}{n} + \frac{(X_i - \bar{X})(X_j - \bar{X})}{S_{XX}}$$
Hence or otherwise, show that the sum of the diagonal elements of $\mathbf{H}$ (called the trace of $\mathbf{H}$) is 2. [Hint: note that the $(i, j)$th term of $\mathbf{H}$ is the $i$th row of $\mathbf{X}$ times $(\mathbf{X}'\mathbf{X})^{-1}$ times the $j$th column of $\mathbf{X}'$]

5. Let $\mathbf{Y} = \mathbf{A}\mathbf{X}$ where $\mathbf{X} = (X_1, X_2)'$ and $\mathbf{A}$ is a $2 \times 2$ matrix of constants.
   (a) Derive the elements of the vector $\mathbf{Y} = (Y_1, Y_2)'$ by matrix multiplication.
   (b) Noting that the covariance matrix of $\mathbf{Y}$ is
   $$cov(\mathbf{Y}) = \begin{pmatrix} var(Y_1) & cov(Y_1, Y_2) \\ cov(Y_1, Y_2) & var(Y_2) \end{pmatrix}$$
   and the result in (a), derive $cov(Y_1, Y_2)$ and hence complete the covariance matrix.
   (c) Show by matrix multiplication that $cov(\mathbf{Y}) = \mathbf{A}cov(\mathbf{X})\mathbf{A}'$

6. In the bivariate regression model, show that the first element of $\boldsymbol{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$ is equal to $\bar{Y} - \hat{\beta}\bar{X}$, where $\hat{\beta}$ is the OLS slope estimator, given by
$$\hat{\beta} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$$
i.e. show that
$$\frac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{n \sum (X_i - \bar{X})^2} = \bar{Y} - \hat{\beta}\bar{X}$$
(see lecture page 136)

7. Suppose $X_1, \ldots, X_n$ are iid geometric random variables with parameter $p$, and probability function $f(x) = (1-p)^{x-1}p$ for $x = 1, 2, \ldots$. Find the maximum likelihood estimator for $p$. Show that this is not equal to the method of moments estimator for $p$. [Hint: see Part1A notes for the mean of the geometric.]